Substituting this value of x in the last two fractions,

$$\frac{dz}{a}=\frac{dy}{1+cy}, \text{ or } z=\frac{a}{c}\log(1+cy)+c'.$$

Replacing c by $\frac{1}{x}-\frac{1}{y}$, $z=\frac{axy}{y-x}\log\frac{y}{x}+\phi\left(\frac{1}{x}-\frac{1}{y}\right)$.

Also solved similarly by J. Scheffer, and G. B. M. Zerr.

293. Proposed by V. M. SPUNAR, M. and E. E., East Pittsburg, Pa.

Find the length of the integral curve of the differential equation $(y^2x^{\frac{4}{3}}+2)dx-x^{\frac{4}{3}}\,dy=0$ between $x_1=1$ and $x_2=8$.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

Let $y=-1/z$, $x^{-\frac{1}{3}}=v$, then the equation becomes $dz+6z^2dv+3v^{-4}dv=0$.

Now, let $z=\frac{1}{6v}+\frac{u}{v^2}$.

Then $v^2du+6u^2dv+3dv=0$, or $\frac{du}{3+6u^2}=-\frac{dv}{v^2}$.

$$\therefore \frac{1}{3\sqrt{2}}\tan^{-1}(u\sqrt{2})=\frac{1}{v}+a=a+x^{\frac{1}{3}}\ldots(1).$$

$$\therefore u=\frac{1}{\sqrt{2}}\tan[3\sqrt{2}(a+x^{\frac{1}{3}})]\ldots(2).$$

$$\therefore y=-\frac{6\sqrt{2}}{6x^{\frac{2}{3}}\tan[3\sqrt{2}(a+x^{\frac{1}{3}})]+x^{\frac{1}{3}}\sqrt{2}} \text{ is the equation.}$$

From (1), $u=\frac{1}{\sqrt{2}}\tan\left[\frac{3\sqrt{2}(av+1)}{v}\right]$.

$$S=\int\sqrt{[1+(du/dv)^2]}\,dv=\int_{\frac{1}{2}}^{1}\frac{1}{v^2}\sqrt{v^4+9\sec^4\left[\frac{3\sqrt{2}(av+1)}{v}\right]}\,dv.$$

294. Proposed by C. N. SCHMALL, New York City.

Examine the function, $f(x)=\frac{(x-1)(x-2)}{(x-3)}$ and determine why its *minimum* value is *greater* than its maximum.

Solution by PROF. F. L. GRIFFIN, Ph. D., Williams College.

The derivative, $f'(x)=\frac{x^2-6x+7}{(x-3)^2}$, vanishes for $x=3\pm\sqrt{2}$, changing its sign from + to − for the smaller of these values and from − to + at the larger. Thus the function decreases from its maximum value $f(3-\sqrt{2})$ to its minimum $f(3+\sqrt{2})$, *except that it passes through* ∞ at $x=3$, and thus it is possible for the minimum to exceed the maximum. The same fact holds for the more general function of the same type: $F(x)=\frac{(x-a)(x-b)}{x-c}$, if $c>a$ and $c>b$; but the maximum of $F(x)$ exceeds the minimum if $c<a$ and $c<b$. Thus the property proposed for explanation depends not so much on the mere numerical values, nor even on the fact of an infinite discontinuity, as upon the order of the zeros of the numerator and denominator.

Also solved by S. G. Barton, J. E. Sanders, V. M. Spunar, and J. Scheffer.

MECHANICS.

245. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

A body moves with constant speed in the circumference of an ellipse. Find the rate of approach (1) to the center, (2) to one of the foci, for any point in the ellipse.

Solution by PROF. F. L. GRIFFIN, Ph. D., Williams College.

Differentiating $b^2x^2+a^2y^2=a^2b^2$ with respect to the time, we have $\frac{dx}{dt}/a^2y=-\frac{dy}{dt}/b^2x, =-\lambda$ (say), whence if k denote the constant speed, $k^2=\left(\frac{ds}{dt}\right)^2=\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2=\lambda^2[a^4y^2+b^4x^2]$, or $\lambda=k/\sqrt{[a^4y^2+b^4x^2]}$. The negative sign should be taken if the motion is clockwise.

Hence, $\frac{dx}{dt}=-ka^2y/\sqrt{[a^4y^2+b^4x^2]}$ and $\frac{dy}{dt}=kb^2x/\sqrt{[a^4y^2+b^4x^2]}$.

(I) Now the radius vector from the center to (x, y) is given by $r^2=x^2+y^2$; whence the rate of approach to the center is

$$-\frac{dr}{dt}=-\left(x\frac{dx}{dt}+y\frac{dy}{dt}\right)/r=-\frac{kxy(b^2-a^2)}{r\sqrt{[a^4y^2+b^4x^2]}}=\frac{ka^2e^2xy}{\sqrt{[(a^4y^2+b^4x^2)(x^2+y^2)]}}.$$

(II) The radius vector from the left-hand focus $(-ae, 0)$ to (x, y) is given by: $r^2=(x+ae)^2+y^2$; whence

$$-\frac{dr}{dt}=-\left[(x+ae)\frac{dx}{dt}+y\frac{dy}{dt}\right]/r=\frac{ka^2ey(ex+a)}{\sqrt{[a^4y^2+b^4x^2)(x^2+y^2)]}}.$$